Foundations of Modern Cosmology

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Abstract

A number of key elements in modern cosmology are explained, starting with the cosmological principle and Hubble's law, followed by concepts like the world map and the world picture and the basic laws governing them. A few caveats are subsequently discussed with the aim to avoid common misinterpretations.

Contents

1 View on the cosmos

1.1 The Copernican principle

Prior to the 16th century, people believed that the Earth was at the center of creation and that all celestial objects revolved around the Earth. Nicolaus Copernicus[1] was the first to realize that the

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[1] Nicolaus Copernicus was a Polish mathematician and astronomer who lived from 1473 till 1543.

stars are located very far away and that the Earth and other planets are in orbit around the Sun. As such, he dismissed the geocentric view on the heavens and replaced it with a heliocentric model. As our understanding of the universe improved over time, the basic thought that we are not privileged observers gained momentum and became known as the Copernican principle.

1.2 The great debate

Nowadays, it is readily accepted that the Milky Way is just a galaxy like many others in the universe. People often don't realize that this insight took shape not longer than about 100 years ago. At the beginning of the previous century, a particular kind of celestial objects called "spiral nebulae" were well known to astronomers. In fact, the Messier Catalogue, published in the 18th century, already contained some of them like the Andromeda galaxy (M31). Nevertheless, early 20th century, these nebulae were a subject of debate, revolving around their true nature and distance.

Herber Curtis[2] was part of the group of astronomers who advocated that spiral nebulae are "island universes" far outside the Milky Way but comparable in size and nature to our own galaxy. Harlow Shapley[3] on the other hand was convinced that spiral nebulae were part of the Milky Way, given that his measurements resulted in a size of the Milky Way far larger than previously thought. The discussion even led to a physical debate between Curtis and Shapley, held in 1920 at the Smithsonian Museum of Natural History.

The outcome of the debate was inconclusive and the astronomical community had to wait until 1924 when Edwin Hubble found that the distance to the Andromeda galaxy is larger than the size of our galaxy. Even though Curtis' ideas therefore turned out to be more accurate, Shapley contributed to our understanding that the Sun's location in our galaxy is by no means special.

1.3 The cosmological principle

In modern cosmology, the Copernican principle is taken a step further in what is called the cosmological principle:

> The universe is homogeneous and isotropic.

A homogeneous universe is translation invariant, meaning that its appearance and properties are everywhere the same. An isotropic universe is rotation invariant and is therefore the same in every direction. Obviously, at the scale of a planetary system or a galaxy, the universe is neither homogeneous nor isotropic. Cosmologists typically use a threshold of 100 Mpc as the scale beyond which local differences are sufficiently averaged out to achieve a homogeneous and isotropic condition.

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[2] Herber Curtis was an American astronomer who lived from 1872 till 1942.

[3] Harlow Shapley was an American astronomer who lived from 1885 till 1972.

2 Hubble's law

2.1 Doppler shift

The Doppler effect is a well-known phenomenon for sound waves but equally affects electromagnetic radiation. When a source moves towards an observer, the emitted waves are compressed and their wavelength gets shorter. A shorter wavelength is equivalent with a higher frequency which explains the high pitch of an approaching ambulance's siren or an approaching train's whistle. Once past the observer, the emitted waves are stretched out and their wavelength consequently gets longer. The equivalent lower frequency makes the pitch of the ambulance's siren or the train's whistle drop when the ambulance or train are passing by.

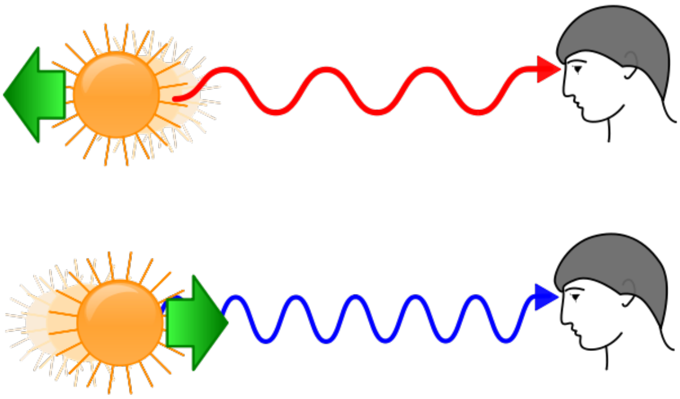

Figure 1: Doppler effect causing a red shift or blue shift in the light of moving objects.

The light of stars and galaxies contains spectral lines which are characteristic for their chemical composition. Compared to the spectral lines measured in a laboratory, those of celestial objects are observed at longer or shorter wavelengths. When they are shifted to the longer wavelength side of the spectrum, the light is called redshifted. In the opposite case, it is said to be blueshifted. In 1912, Vesto Slipher[4] was the first astronomer who observed the Doppler shift in the light of what were called faint nebulae at that time in history. He discovered that, apart from a few exceptions, they all exhibit a redshift and are thus receding away from us.

To quantify the amount of shift in an observed spectrum, physicists use the dimensionless parameter z. In terms of the emitted wavelength λ_{em} and the observed wavelength λ_{obs} it is defined as:

$$z \equiv \frac{\lambda_{obs} - \lambda_{em}}{\lambda_{em}}$$

Note that z is positive in case of redshift en negative in case of blueshift and is therefore commonly referred to as redshift. Rearranging the terms leads to the alternative definition:

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[4] Vesto Slipher was an American astronomer who lived from 1875 till 1969.

$$\boxed{z + 1 \equiv \frac{\lambda_{obs}}{\lambda_{em}}} \tag{1}$$

If v is the radial velocity of the source relative to the observer and c is the speed of light, then the ratio of λ_{obs} and λ_{em} according to the theory of special relativity is given by the relativistic Doppler formula:

$$\boxed{\frac{\lambda_{obs}}{\lambda_{em}} = \frac{\sqrt{1 + v/c}}{\sqrt{1 - v/c}}} \tag{2}$$

For velocities which are small compared to the speed of light[5], the classic Doppler formula is derivable from equation (2) by using the binomial theorem for both the numerator and denominator:

$$\frac{\lambda_{obs}}{\lambda_{em}} \approx \left(1 + \frac{1}{2}\,\frac{v}{c}\right)\left(1 + \frac{1}{2}\,\frac{v}{c}\right)$$

$$\boxed{\frac{\lambda_{obs}}{\lambda_{em}} \approx 1 + \frac{v}{c}} \tag{3}$$

Combining equations (1) and (3) gives the approximate velocity-redshift relation used by Vesto Slipher to work out the radial velocity of the nebulae he observed:

$$v \approx c\,z \tag{4}$$

2.2 Hubble's discovery

At the beginning of the 19th century, the vastness of the universe was not yet known and space was thought to be not much bigger than the Milky Way. When Edwin Hubble[6] started measuring the distance of faint nebulae in 1922, his observations showed that they are located so far away that they cannot be part of the Milky Way. Only then, astronomers realized that the faint nebulae are galaxies themselves, located far away from us.

In 1929, Edwin Hubble combined his work with the work of Vesto Slipher and discovered a roughly linear relation between redshift z and distance r:

$$c\,z = H\,r$$

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[5] $v << 0.01\,c$

[6] Edwin Hubble was an American astronomer who lived from 1889 till 1953.

Using approximation (4) this easily translates into the velocity-distance law known as Hubble's law:

$$v = H\,r \tag{5}$$

The proportionality factor between velocity v and distance r is called the Hubble parameter H. The Hubble parameter is time dependent which means that it is not necessarily constant over time. The term "Hubble constant" therefore only refers to the present value of the Hubble parameter $H(t_0)$. Figure 2 is a copy of the figure which appeared in Hubble's original paper.

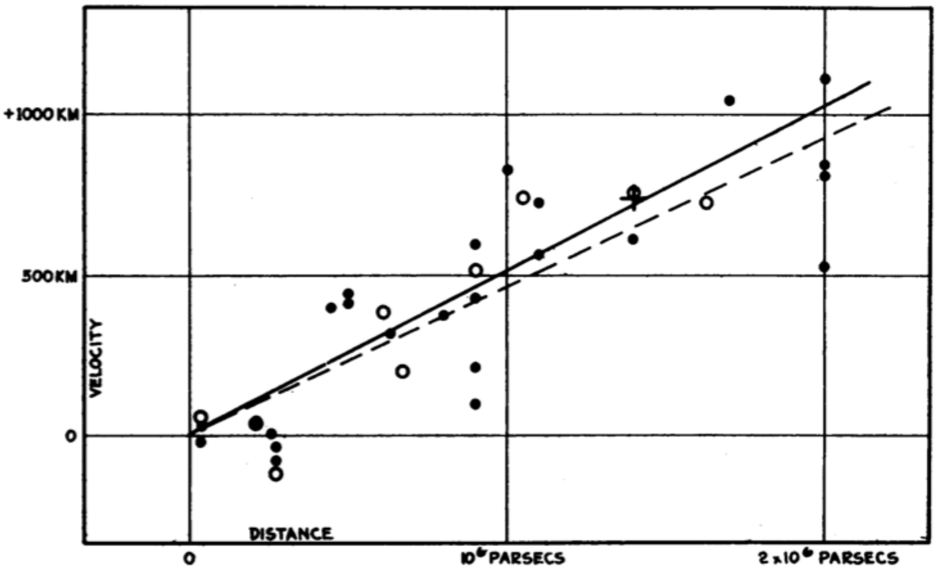

Figure 2: Velocity-distance relation among extra-galactic nebulae as it appeared in Hubble's original paper dated 1929.

In the next section, it will be shown that velocity-distance law (5) is universally true for any expanding (or receding) homogeneous and isotropic universe. Slipher and Hubble were lucky that the nebulae they had been observing were not too distant from Earth so that they could use velocity-redshift relation (4) without introducing large errors. If not, there would not have been a linear relation between redshift and distance in their data and Hubble might not have observationally discovered the law which bears his name.

2.3 Expansion redshift

While the Doppler shift observed in the light (or sound) of a moving source is a good way to illustrate the redshift observed in the light of distant galaxies, their redshift is strictly speaking caused by something else. The Doppler effect is the result of the relative motion between an emitter and an observer at the time radiation is emitted and/or observed and is governed by the rules of special relativity. This is not the case for galaxies:

Galaxies are not moving through space but are moving apart with space.

As such, the redshift in their light is caused by the expansion of space and governed by the laws of general relativity. In other words, recession velocities are no velocities like those defined by Newton's laws or special relativity and consequently:

Recession velocities are not constrained by the speed of light.

To illustrate why the Doppler effect is not at play, consider the following thought experiment. Imagine a galaxy which is stationary compared to Earth and emits a pulse of light. Once the pulse is emitted, the galaxy starts moving through space, receding from the earthly observer. Before the pulse arrives, the motion stops. The pulse of light will arrive without redshift as there was no relative motion between source and observer, neither at the time of emission nor at the time of observation.

Next imagine the same stationary galaxy emitting a pulse of light. Now space starts expanding once the pulse is emitted and stops expanding before the pulse arrives. In this case, the light arrives with a redshift as its wavelength is influenced by expanding space, even if expansion only starts after the light's emittance and stops before arrival of the pulse.

The above shows that in fact, Slipher and Hubble were not only lucky that the recession velocities of the nebulae they observed were relatively small so that the approximation derived from the classic Doppler formula and given by equation (4) could be applied. They were also lucky that the application of the Doppler formula for a phenomenon which is not caused by the Doppler effect, did not yield meaningless results.

3 Basic laws

3.1 Velocity-distance law

Consider a coordinate system of which the unit vectors expand along with space. Distant galaxies are stationary with respect to such a co-moving coordinate system, their peculiar motion not taken into account. The co-moving coordinate distance between any 2 galaxies is therefore a constant. If the expansion of space is homogeneous and isotropic, i.e. independent of location and direction, the true distance r and the coordinate distance r_c are related to each other by a scale factor which is everywhere the same and independent of any spatial coordinate. The scale factor a then only depends on time and the relation between both distances is given by:

$$r = a\,r_c \tag{6}$$

Velocity is the change of distance per unit of time and differentiating equation (6) with respect to time consequently yields:

$$v = \frac{dr}{dt} = \frac{da}{dt}\, r_c \tag{7}$$

Combining equations (6) and (7), eliminating the constant r_c and introducing the dot notation for time derivatives results in:

$$v = \frac{\dot{a}}{a}\, r \tag{8}$$

A convenient way to define the Hubble parameter H based on the scale factor a is:

$$\boxed{H \equiv \frac{\dot{a}}{a}} \tag{9}$$

This not only shows that the Hubble parameter is everywhere the same in a homogeneous and isotropic universe and only dependent on time, given that the same applies for the scale factor it is defined with. At the same time, this definition transforms equation (8) into the velocity-distance relation previously encountered as Hubble's law (5):

$$\boxed{v = H\, r} \tag{10}$$

Equation (10) is rigorously true in the world map and only true at short distances in the world picture. The world map depicts how a theorist visualizes the universe at any given instant of cosmic time, while the world picture reflects how the universe is perceived by an observer who is part of that universe. Due to the fact that the speed of light is finite, the world picture is a mix of images dating from different era backward in time. Figure 3 illustrates both concepts.

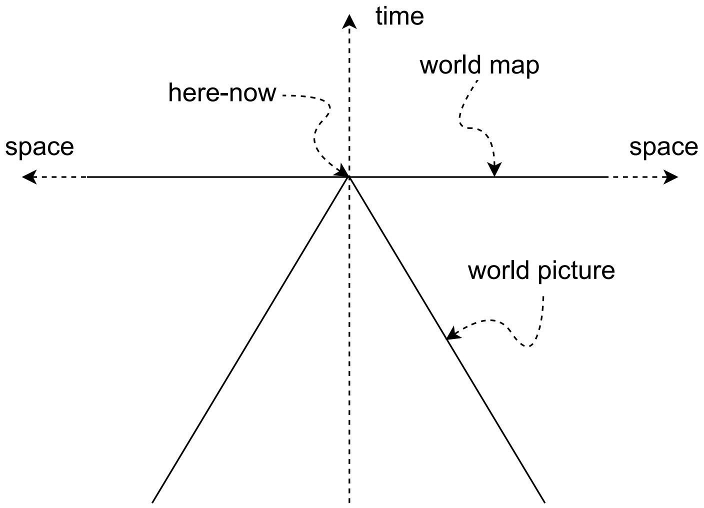

Figure 3: Schematic representation of the world picture and world map of a space-time with only 1 spatial dimension. The world picture is how the universe is observed here and now and is heavily affected by the finite speed of light. The world map is how the universe would be seen here and now by a god-like creature who can instantaneously oversee the entirety of space.

3.2 Expansion-redshift law

In an expanding universe, light waves are stretched out along with the space they are travelling through. The ratio between the emitted wavelength and the observed wavelength is equal to the ratio between the scale factor $a(t) = a$ at time t of emittance and the scale factor $a(t_0) = a_0$ at time t_0 of observation:

$$\frac{\lambda_{obs}}{\lambda_{em}} = \frac{a_0}{a} \tag{11}$$

Combining equations (1) and (11) gives the important expansion-redshift law:

$$\boxed{z + 1 = \frac{a_0}{a}} \tag{12}$$

It tells us how much the universe has expanded since the radiation of a distant object was emitted. If e.g. a quasar is observed to have a redshift of 0.37, the universe has become 1.37 times larger or has increased by $37\ \%$ since the radiation left the quasar.

4 Caveats

4.1 Measuring recession velocities

One of the pitfalls in cosmology is the misconception that the relativistic Doppler formula (2), or for worse the classic Doppler formula (3), allow converting an observed cosmological redshift to a recession velocity. This is very tempting as recession velocities are generally more appealing to the general public than cosmological redshifts, but incorrect as expansion redshifts are no Doppler shifts.

4.2 Superluminal recession velocities

Remember that the universe does not expand in space but instead, consists of expanding space. Galaxies do not move through space but instead, float stationary in space with respect to a co-moving coordinate system. Separating distances increase because the space between galaxies expands. Recession velocities are consequently governed by general relativity and therefore fundamentally different from Newtonian velocities or velocities governed by special relativity. As such, they are not constrained by the speed limit imposed by special relativity, namely the speed of light.

4.3 Measuring the Hubble constant

Given that velocity-distance law (10) is only rigorously valid in the world map while expansion-redshift law (12) applies to observations in the world picture, measuring H_0 requires a way to relate observations in the world picture to the world map. More fundamentally, a theory about how the scale factor a changes with time, i.e. how the universe evolves, is needed. In practise, there is no straightforward way to achieve such a projection of the world picture onto the world map as there are different possibilities for the geometry and evolution of the universe.

Only when distances are small, there is a way to combine world map and world picture into a redshift-distance law. In that limiting case, it is acceptable to rewrite equations (9) and (12) as difference equations:

$$H = \frac{\Delta a}{a\,\Delta t}$$

$$z = \frac{a_0 - a}{a} = \frac{\Delta a}{a}$$

Combined they lead to:

$$z = H\,\Delta t \tag{13}$$

At the same time, small distances are well approximated by the light travel time multiplied by the speed of light:

$$r = c\,\Delta t \tag{14}$$

Combining equations (13) and (14) finally gives:

$$c\,z = H\,r$$

A careful reader will notice this is exactly the linear redshift-distance law discovered by Edwin Hubble in his observational data.