High School Mathematics Review

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Abstract

Frequently encountered high school level mathematical formulae are listed as a memory aid. No further proof or derivation is given and the list is by no means intended to be comprehensive.

Contents

1 Surface areas and volumes

1.1 Spheres

The surface area A of a sphere with radius r is given by:

$$A = 4\,\pi\,r^2$$

The volume V of a sphere with radius r is given by:

$$V = \frac{4}{3}\,\pi\,r^3$$

2 Basic calculation rules

2.1 Special products

$$(a+b)^2 = a^2 + b^2 + 2\,a\,b$$

$$(a-b)^2 = a^2 + b^2 - 2\,a\,b$$

2.2 Trigonometry

$$\cos^2 a + \sin^2 a = 1$$

$$\cos\,(a \pm b) = \cos\ a\ \cos\ b \pm \sin\ a\ \sin\ b$$

$$\sin\,(a \pm b) = \sin\ a\ \cos\ b \pm \cos\ a\ \sin\ b$$

$$\cos\ a\ \cos\ b = \frac{1}{2}\,[\cos\,(a+b) + \cos\,(a-b)]$$

$$\sin\ a\ \sin\ b = \frac{1}{2}\,[\cos\,(a-b) - \cos\,(a+b)]$$

$$\sin\ a\ \cos\ b = \frac{1}{2}\,[\sin\,(a+b) + \sin\,(a-b)]$$

2.3 Hyperbolic trigonometry

$$\cosh^2 a - \sinh^2 a = 1$$

$$\cosh\,(a \pm b) = \cosh\ a\ \cosh\ b \pm \sinh\ a\ \sinh\ b$$

$$\sinh\,(a \pm b) = \sinh\ a\ \cosh\ b \pm \cosh\ a\ \sinh\ b$$

$$\cosh\ a\ \cosh\ b = \frac{1}{2}\,[\cosh\,(a+b) + \cosh\,(a-b)]$$

$$\sinh\ a\ \sinh\ b = \frac{1}{2}\,[\cosh\,(a+b) - \cosh\,(a-b)]$$

$$\sinh\ a\ \cosh\ b = \frac{1}{2}\,[\sinh\,(a+b) + \sinh\,(a-b)]$$

2.4 Logarithms

$$\log_h (a\,b) = \log_h a + \log_h b$$

$$\log_h \left(\frac{a}{b}\right) = \log_h a - \log_h b$$

$$\log_h a^n = n \log_h a$$

$$\log_h \sqrt[n]{a} = \frac{\log_h a}{n}$$

$$\log_h h^n = n$$

$$\log_h a = \frac{\log_k a}{\log_k h}$$

Note that when the base of $\log_h a$ is the natural number (or Euler's number) e, the corresponding logarithm is called the natural logarithm and written as $\ln a$.

2.5 Exponentiation

$$a^n\,a^m = a^{n+m}$$

$$\frac{1}{a^n} = a^{-n}$$

$$(a^n)^m = a^{n\,m}$$

$$(a\,b)^n = a^n\,b^n$$

3 Derivative of a function

3.1 Definition

If f is a function of variable x, the derivative of f with respect to x is defined as:

$$\frac{df}{dx} = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

Note that it is common practise to write the time derivative of a function $x(t)$ as $\dot{x}$.

3.2 Calculation rules

The following calculation rules apply for operations with functions $f(x)$ and $g(x)$:

$$\frac{d}{dx}(f\ g) = g\ \frac{df}{dx} + f\ \frac{dg}{dx}$$

$$\frac{d}{dx}\left(\frac{f}{g}\right) = \frac{g\ \frac{df}{dx} - f\ \frac{dg}{dx}}{g^2}$$

$$\frac{d}{dx}(f^n) = n\ f^{n-1}\ \frac{df}{dx}$$

3.3 Trigonometric functions

$$\frac{d}{dx}\cos\left(a\ x\right) = -a\ \sin\left(a\ x\right)$$

$$\frac{d}{dx}\sin\left(a\ x\right) = a\ \cos\left(a\ x\right)$$

3.4 Hyperbolic functions

$$\frac{d}{dx}\cosh\left(a\ x\right) = a\ \sinh\left(a\ x\right)$$

$$\frac{d}{dx}\sinh\left(a\ x\right) = a\ \cosh\left(a\ x\right)$$

3.5 Logarithmic functions

$$\frac{d}{dx}\log_h\left(a\ x\right) = \frac{1}{x\ \ln\ h}$$

$$\frac{d}{dx}\ln\left(a\ x\right) = \frac{1}{x}$$

3.6 Exponential functions

An exponential function is a function $f(x)$ in which the function argument x appears as exponent.

$$\frac{d}{dx}h^{a\ x} = a\ h^{a\ x}\ \ln\ h$$

$$\frac{d}{dx}e^{a\ x} = a\ e^{a\ x}$$

3.7 Power functions

A power function is a function $f(x)$ in which the function argument x appears as base.

$$\frac{d}{dx}x^n = n\ x^{n-1}$$

4 Theorems

4.1 Pythagoras' theorem

If A and B are the perpendicular sides of a right-angled triangle in plane geometry and C the side opposing the right angle, then Pythagoras' theorem states:

$$C^2 = A^2 + B^2$$

The same is true in Euclidean space and can be used to work out the magnitude $|v|$ of a vector $\vec{v}$ with vectorial components (v_x, v_y, v_z) along 3 perpendicular axis. According Pythagoras' theorem, the vector's magnitude is then:

$$|v|^2 = v_x^2 + v_y^2 + v_z^2$$

4.2 Binomial theorem

For any value of n, whether positive, negative, integer or non-integer, the value of the n-th power of binomial $(a + b)$ is given by:

$$(a+b)^n = a^n + n\, a^{n-1}\, b + \frac{n\,(n-1)}{2}\, a^{n-2}\, b^2 + ... + b^n$$

$$(a+b)^n = \sum_{k=0}^{n} \frac{n!}{(n-k)!\, k!}\, a^{n-k}\, b^k$$

When $b << a$, the first few terms provide a good approximation.