The Muon Mystery

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Abstract

Showers of muons are produced in Earth's upper atmosphere as a result of collisions between cosmic ray particles and air molecules. Historic experiments involving cosmic ray muons have demonstrated that the number of muons reaching Earth's surface exceeds the expected number based on their average lifetime. After an introduction to Einstein's theory of special relativity, it is shown how special relativity lifts the veil of the discrepancy and how muon experiments actually confirm Einstein's theory.

Contents

1 Cosmic ray muons

1.1 Origin

Earth's atmosphere is constantly bombarded with energetic particles traveling through space at nearly the speed of light. These cosmic rays not only consist of photons but, contrary to what their name suggests, also include matter-like particles like hydrogen and helium nuclei. Low energy cosmic rays originate from the Sun and reach Earth with the Solar wind, the continuous stream of particles released in space by the Sun. The source of high energy cosmic rays was long time unknown but in the past years, astrophysicists have identified several sources which are capable of generating high energy particles.

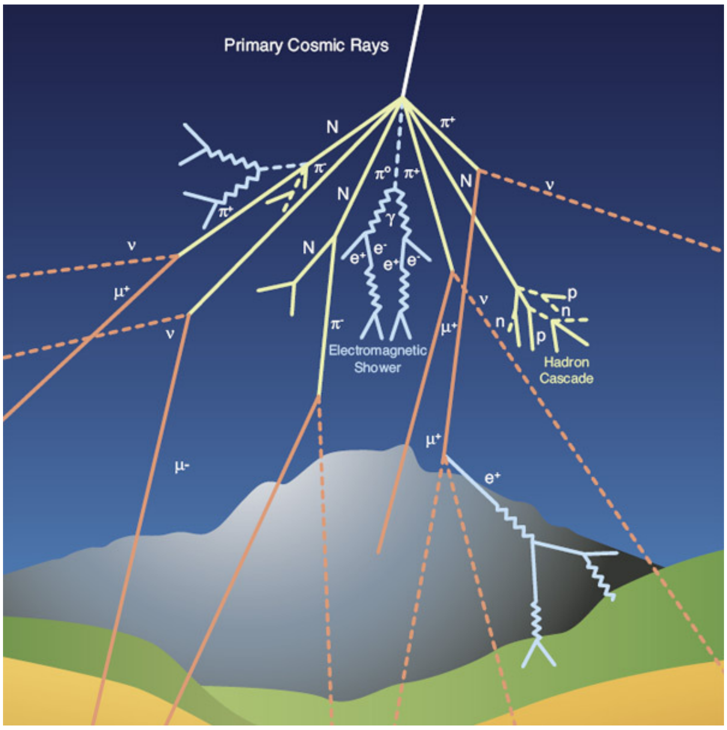

Figure 1: Shower of secondary particles resulting from a collision between a cosmic ray particle and a molecule in Earth's atmosphere. (Credit: Marzena Lapka, CERN)

A cosmic ray particle colliding with a molecule in Earth's atmosphere gives rise to a shower of secondary particles which continue their journey at nearly the speed of light. One type of particle formed that way is the muon. According the Standard Model of Particle Physics, muons and electrons belong to the leptons, elementary particles which have a half-integer spin and which do not undergo strong interactions. The rest mass of a muon is larger than that of an electron but their charge is the same. A muon has an average lifetime τ_μ of $2.2\ \mu\text{s}$ after which it spontaneously decays into other elementary particles, usually an electron and two neutrinos.

1.2 The muon mystery

Muon decay follows an exponential law $N(t)$ which gives the remaining quantity after a time t, if N_0 is the initial quantity:

$$N(t) = N_0\, e^{-t/\tau_\mu} \tag{1}$$

Figure 2 shows how the remaining population N/N_0 evolves over time according to decay law (1) for a particle with an average lifetime of $2.2\ \mu\text{s}$ like the muon, as compared to imaginary particles with average lifetimes 10 and 5 times larger.

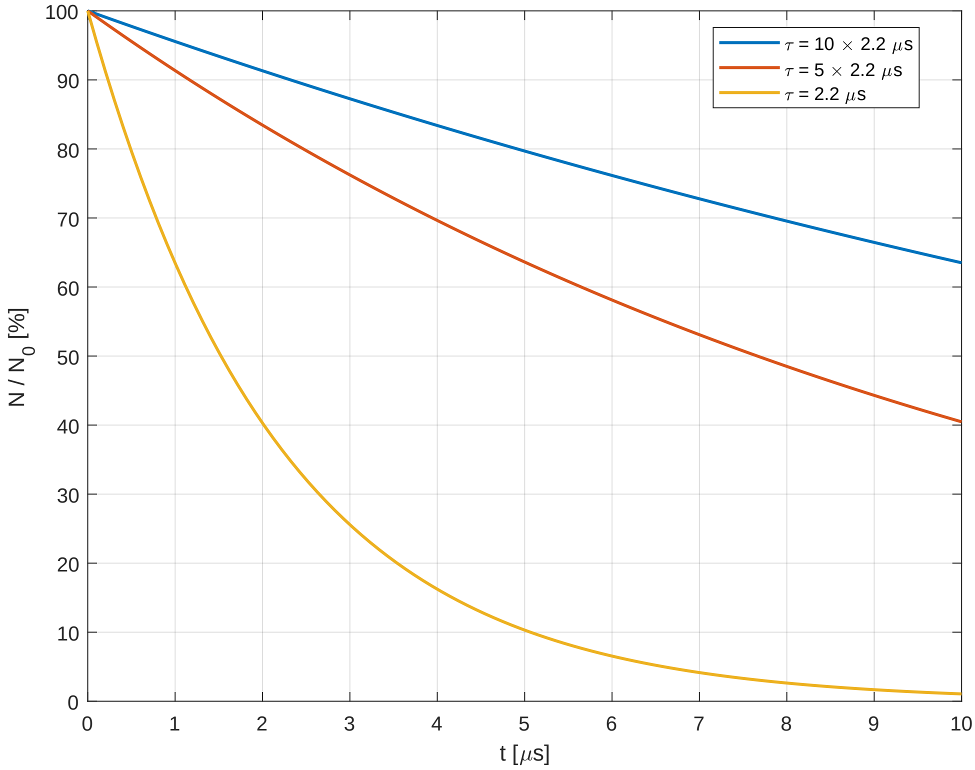

Figure 2: Remaining percentage of the initial population as a function of time, for a particle with an average lifetime of $2.2\ \mu s$ like the muon, compared to imaginary particles with average lifetimes 10 and 5 times larger.

Experiments have compared the number of muons observed at a given altitude to the number detected at sea level. One such experiment[1] counted 563 muons per hour at the top of Mt. Washington, $1907\ \mathrm{m}$ above sea level, rushing down at $99.5\ \%$ of the speed of light. At such speed, it takes the muons about $6.39\ \mu s$ to travel $1907\ \mathrm{m}$. According to decay law (1), the fraction of muons left by the time they reach sea level is then:

$$N(6.39\mu s)/N_0 \approx e^{-6.39/2.2}$$

$$\approx 5.5\%$$

For an initial quantity of 563, this is about 31 muons per hour. Surprisingly, the experiment measured 408 muons per hour at sea level, which is more than a tenfold!

[1] David Frisch, James Smith, *Measurement of the Relativistic Time Dilation Using μ-Mesons*, American Journal of Physics, 1963

2 Special relativity

2.1 Time dilation

Imagine a clock which measures time by means of a repeating light pulse, installed on board a maglev train moving at a constant speed v. The light pulse is fired vertically upward from a laser diode, hits a mirror positioned at a certain distance H above the emitting diode and reflects vertically downward to a detector mounted next to the laser diode. Figure 3 illustrates the concept.

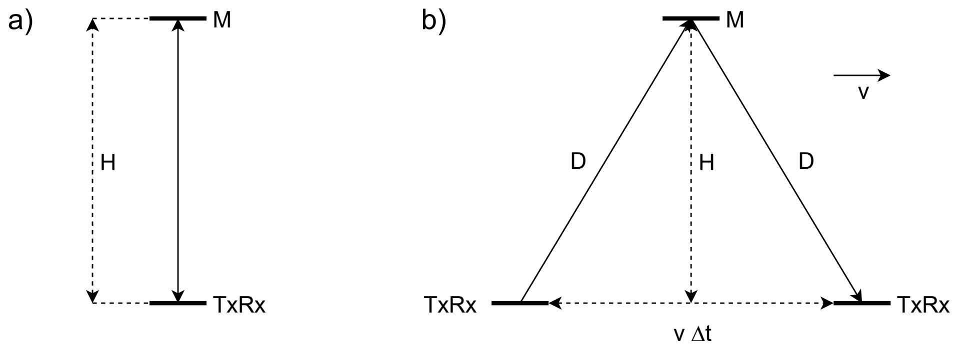

Figure 3: Schematic representation of a clock which measures time with a repeating light pulse bouncing back on an overhead mirror. Part a) depicts how the light's trajectory is seen by an observer who is at rest compared to the clock. Part b) illustrates how the same trajectory is perceived when clock and observer are in relative motion to one another.

If c is the speed of light, the time Δt_{rest} needed to complete one clock cycle as measured by a passenger on the train is:

$$\Delta t_{rest} = \frac{2\,H}{c} \tag{2}$$

The subscript "rest" means that the clock is at rest compared to the passenger's frame of reference.

A bystander along the train's track will see the light pulse follow a triangular path as illustrated in part b) of figure 3 due to the fact that the clock is in motion. The time Δt_{moving} needed to complete one clock cycle as measured by the bystander is:

$$\Delta t_{moving} = \frac{2\,D}{c}$$

$$\Delta t_{moving} = \frac{2}{c}\sqrt{\left(\frac{v\,\Delta t_{moving}}{2}\right)^2 + H^2}$$

The subscript "moving" indicates that the clock is in motion compared to the bystander's frame of reference.

Taking the square of both sides and substituting H from equation (2) leads to:

$$\Delta t_{moving}^2 = \frac{4}{c^2}\left[\left(\frac{v\ \Delta t_{moving}}{2}\right)^2 + H^2\right]$$

$$\Delta t_{moving}^2 = \frac{4}{c^2}\left[\frac{v^2\ \Delta t_{moving}^2}{4} + \frac{c^2\ \Delta t_{rest}^2}{4}\right]$$

$$\left(1 - \frac{v^2}{c^2}\right)\Delta t_{moving}^2 = \Delta t_{rest}^2$$

$$\boxed{\Delta t_{moving} = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}\ \Delta t_{rest}} \qquad (3)$$

This shows that one clock cycle lasts longer for a clock which is in motion compared to the observer than an identical clock at rest:

$$\Delta t_{moving} \geq \Delta t_{rest}$$

In other words:

Moving clocks run slow.

This phenomenon is called ”time dilation” and is a consequence of the fact that the speed of light is the same in every reference frame. Time dilation only reveals itself when time measurements are compared. The shortest time is observed in a reference frame at rest with respect to the clock. This is the ”proper time”.

The discrepancy gets larger as the relative speed v between reference frames increases and the proportionality factor is known as the Lorentz factor γ:

$$\boxed{\gamma = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}} \qquad (4)$$

Figure 4 shows how the Lorentz factor increases with the ratio v/c, starting at 1 when the speed is nil and tending to infinity as the speed of light is approached.

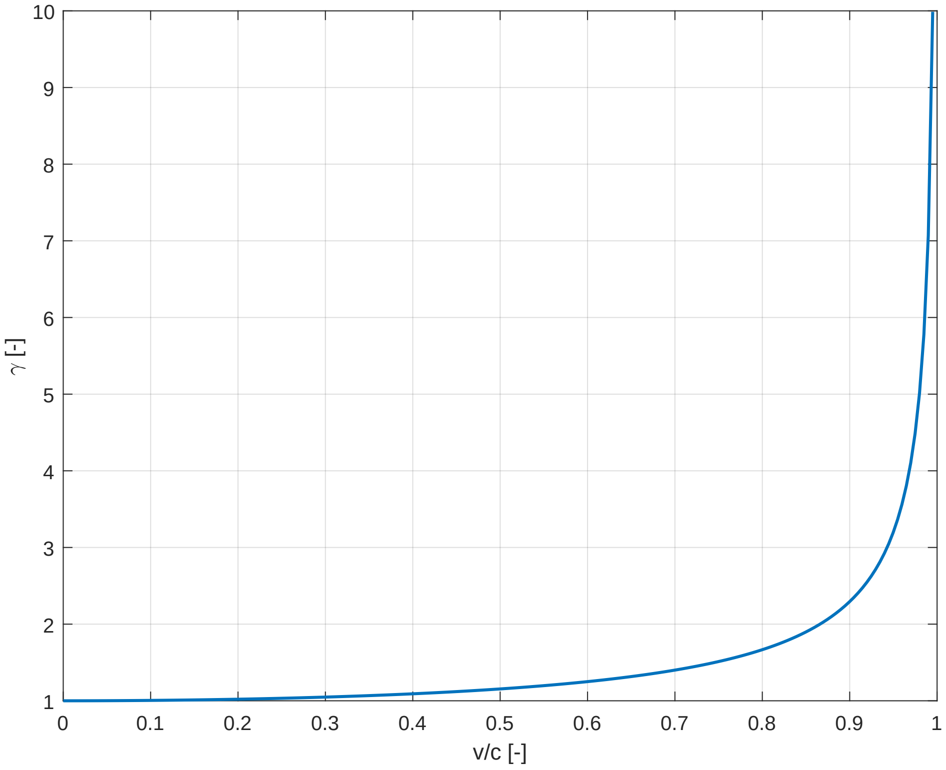

Figure 4: Lorentz factor as a function of the relative speed between reference frames, expressed as a fraction of the speed of light.

2.2 Length contraction

Now imagine there are light posts installed along the train's track at regular distances from one another and that both a passenger aboard the maglev train and a bystander along the track wish to measure the distance L between 2 consecutive light poles. The passenger, for whom the poles are moving at the train's speed v, uses the moment (s)he sees a pole passing as zero reference and measures the time, using the clock aboard the train, until the next light pole passes. (S)he will find:

$$L_{moving} = v\,\Delta t_{rest}$$

The bystander uses the moment the forward edge of the train passes a pole as zero reference and measures the time, using the clock aboard the train, it takes the train to reach the next light pole. (S)he will find:

$$L_{rest} = v\,\Delta t_{moving}$$

Eliminating v from both equations yields:

$$\frac{L_{moving}}{L_{rest}} = \frac{\Delta t_{rest}}{\Delta t_{moving}}$$

Combining with equation (3) gives:

$$\boxed{L_{moving} = \sqrt{1 - \left(\frac{v}{c}\right)^2}\, L_{rest}}$$

This shows that the same distance measures longer in the reference frame of the bystander than in the reference frame of the passenger:

$$L_{rest} \geq L_{moving}$$

In other words:

Moving objects get shorter.

This phenomenon is known as ”length contraction”. The longest distance is measured in a reference frame at rest. This is the ”proper length”.

2.3 Spacetime metric

Events in our daily lives are characterized by the point in time at which they occur and the location where they occur. Practically speaking, each event is associated with a time coordinate t and 3 spatial coordinates x, y, and z in 3-dimensional space. Together, these coordinates form a 4-dimensional spacetime. The spacetime separation Δs between 2 events is given by the Minkowski spacetime metric:

$$\boxed{\Delta s^2 = c^2\, \Delta t^2 - \left(\Delta x^2 + \Delta y^2 + \Delta z^2\right)} \tag{5}$$

Depending on the sign of Δs^2, the spacetime separation between events is either space-like, time-like or light-like:

space-like The spacetime separation between 2 events is called ”space-like” when $\Delta s^2 < 0$, i.e. when the spatial term of equation (5) is dominant.

time-like The spacetime separation between 2 events is called ”time-like” when $\Delta s^2 > 0$, i.e. when the temporal term of equation (5) is dominant.

light-like The spacetime separation between 2 events is called ”light-like” when $\Delta s^2 = 0$ as light always follows a path for which $\Delta s^2 = 0$.

To illustrate the above, depict 2 light bulbs which simultaneously emit a flash of light. Both flashes occur with a space-like separation given that $\Delta t = 0$ and that both light bulbs can obviously not be at the same location. Similarly, depict a single light bulb which emits 2 flashes at an interval Δt. If the light bulb remains stationary between the flashes, they now occur with a time-like separation given that $\Delta t \neq 0$.

It is important to understand that the spacetime separation Δs between events is invariant to the reference frame from which the events are observed. This implies that events which occur simultaneously in one reference frame do not necessarily occur simultaneously in another reference frame.

The lights bulbs introduced above, flash at the same time for an observer at rest compared to the bulbs. As discussed, an observer in motion compared to the bulbs experiences a length contraction between them. If Δs has to remain the same regardless of the reference frame, there must be a $\Delta t \neq 0$ for the moving observer to compensate for the length contraction. Consequently, (s)he observes a delay between the flashes.

2.4 The muon mystery revisited

Now back to the cosmic ray muon, with an average lifetime τ_μ of $2.2\ \mu\text{s}$, rushing down Mt. Washington at $99.5\ \%$ of the speed of light. What consequences bears the theory of special relativity?

The Lorentz factor γ as defined in equation (4) in this case equals:

$$\gamma = \frac{1}{\sqrt{1 - (0.99.5)^2}}$$

$$\approx 10.0$$

A stationary observer on Earth's surface experiences the average lifetime of the muons to be $\gamma\ \tau_\mu$ due to time dilation in which τ_μ is the proper average lifetime of the muon, i.e. the average lifetime measured in a laboratory for muons at rest. The decay laws drawn in figure 2 for τ equalling 10 and 5 times τ_μ are therefore not so much the curves for imaginary particles, but for muons travelling at respectively $99.5\ \%$ and $98\ \%$ of the speed of light, given that the Lorentz factor γ for those speeds is respectively about 10 and 5. The fraction of muons left by the time they reach sea level travelling at $99.5\ \%$ of the speed of light according to decay law (1) is then:

$$N(6.39\mu s)/N_0 \approx e^{-6.39/22}$$

$$\approx 74.8\%$$

For an initial quantity of 563, this is about 421 muons per hour which matches the experimentally observed number of 408 muons per hour!

Oppositely, the muons experience Mt. Washington to be less high due to length contraction as the mountain is in motion compared to their rest frame. In that sense, the muons only have to travel a distance of H/γ or $190.7\ \text{m}$ for which they need about $0.639\ \mu\text{s}$ at $99.5\ \%$ of the speed of light. Applying decay law (1) to the travel time and average life time valid in the rest frame of the muons, yields the same result of $74.8\ \%$. This illustrates that the time dilation experienced in one reference frame is equivalent to the length contraction experienced in another reference frame.

The conclusion is that the Mt. Washington experiment and others alike did not yield erratic results, but instead confirmed Einstein's theory of special relativity.