Vertical Aircraft Navigation

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Abstract

Following a short introduction into vertical navigation related terminology, basic formulae and rules of thumb are derived, allowing a pilot of a fixed-wing aircraft to calculate the lateral distance required for a given descent and to calculate the rate at which the airplane needs to descend to remain on the desired glide slope. The reader remains responsible for verifying the correctness of the formulae and graphs in this document and their correct application.

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1 Terminology

1.1 Vertical navigation

Vertical navigation refers to an aircraft's movements in vertical direction and is particularly important during the climb and descent phases of a flight. The full flight trajectory is determined by the combination of vertical and lateral navigation, the latter being an aircraft's movements in the horizontal plane.

This document focuses on the vertical flight profile when approaching the destination airport. It is of utmost importance for a pilot to know at what distance (s)he has to initiate the descent and at what rate. Not only shall the airplane be at the right altitude at the right place to touch down at the start of the runway, there are often altitude constraints to be met at different waypoints along the approach route.

1.2 Vertical distances

In aviation, different terminology is used to denote vertical distances depending on the reference plane which is used to measure the distance from:

Height is an aircraft's vertical distance above ground level and is usually expressed in feet above ground level, abbreviated as "ft AGL". The correct reference plane is entered in the aircraft's altimeters by setting their barometric reference pressure to the local barometric pressure at the airfield, a value commonly known as QFE. On the ground, the aircraft's altimeters then indicate zero.

Altitude is an aircraft's vertical distance above mean sea level and is usually expressed in feet above mean sea level, abbreviated as "ft AMSL". The correct reference plane is entered in the aircraft's altimeters by setting their barometric reference pressure to the local barometric pressure at the airfield reduced to mean sea level, a value commonly known as QNH. On the ground, the aircraft's altimeters then indicate the airfield's elevation above mean sea level.

Flight level is an aircraft's vertical distance above the barometric reference plane of $1013.25\,\mathrm{hPa}$ or $29.92\,\mathrm{in\,Hg}$ and is expressed in hundreds of feet as flight levels (e.g. FL240). The correct reference plane is entered in the aircraft's altimeters by setting their barometric reference pressure to $1013.25\,\mathrm{hPa}$ or $29.92\,\mathrm{in\,Hg}$, a value commonly known as standard pressure. On the ground, the aircraft's altimeters then indicate a value which depends on the difference between standard pressure and the true barometric pressure at the airfield.

The relationship between height, altitude and flight level is illustrated in figure 1.

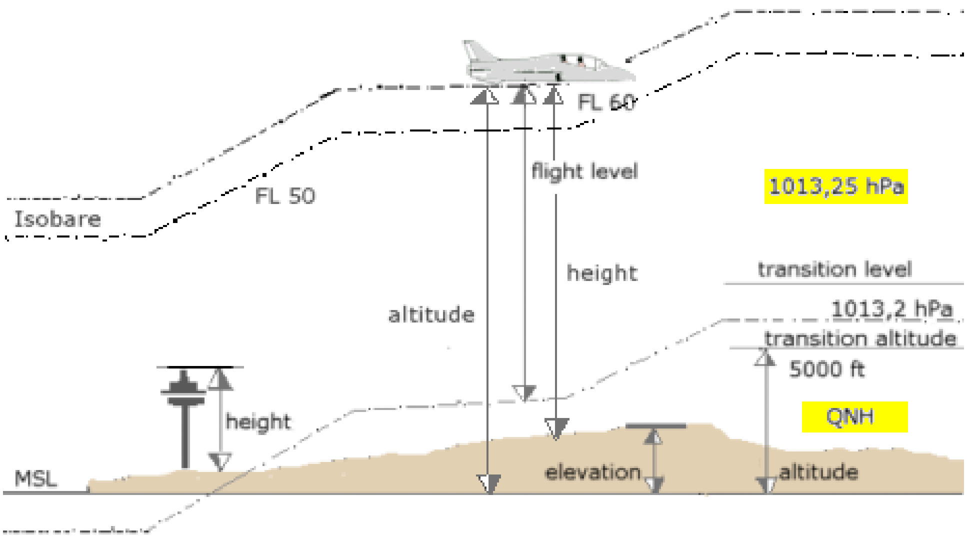

Figure 1: Relationship between height, altitude and flight level. (Credit: Wessmann, Wikimedia Commons)

It would be very inconvenient if the altimeter setting would regularly have to be adjusted to the local barometric pressure reduced to mean sea level while the flight progresses. Airliners therefore transition from altitudes to flight levels as soon as there is no longer a risk of colliding with ground obstacles. In cruise flight, the reference plane is then the same for all aircraft and does not change, no matter how long the flight takes. This makes it much more easier to maintain a safe vertical separation between aircraft. The transition altitude at which the altimeters have to be set to standard pressure in indicated on air navigation charts. During descent, the transition level and QNH to be used for the approach are usually communicated to the cockpit crew by Air Traffic Control.

2 Descent profiles

2.1 Required lateral distance

Figure 2 is a schematic representation of an aircraft's descent profile.

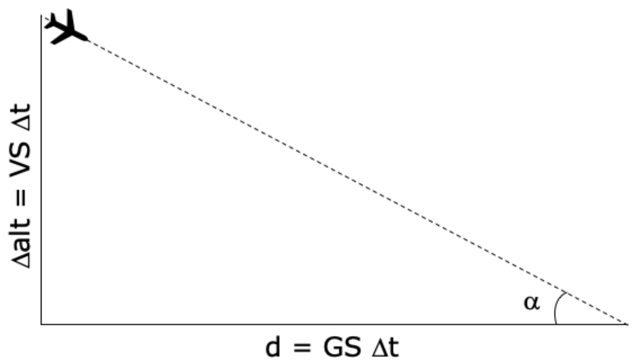

Figure 2: Schematic descent profile in which Δalt is the vertical distance to loose, VS the vertical speed, Δt an arbitrary time interval, d the lateral distance required for the descent, GS the ground speed and α the glide slope angle.

Basic trigonometry yields:

$$\tan\,\alpha = \frac{\Delta alt}{f_1\,d} \tag{1}$$

Factor f_1 in equation (1) is nothing more than a unit conversion factor and approximately equals $6076\ \mathrm{ft/Nm}$ as vertical distances are usually expressed in feet[1] while lateral distances are expressed in nautical miles[2].

Rearranging equation (1) and substituting the value of f_1, the lateral distance d required for the descent expressed as a function of the vertical distance Δalt to loose and the desired glide slope angle α becomes:

[1] $1\ \mathrm{ft} = 0.3048\ \mathrm{m}$
[2] $1\ \mathrm{Nm} = 1852\ \mathrm{m}$

$$d \approx \frac{\Delta alt}{6076\ \tan\ \alpha} \tag{2}$$

Equation (2) is plotted in figure 3 with d as x-axis and Δalt as y-axis for α ranging from $2°$ to $5°$. By entering the graph via the y-axis with the vertical distance to loose and finding the intersection with the line of the desired glide slope angle, the corresponding lateral distance required for the descent is found on the x-axis.

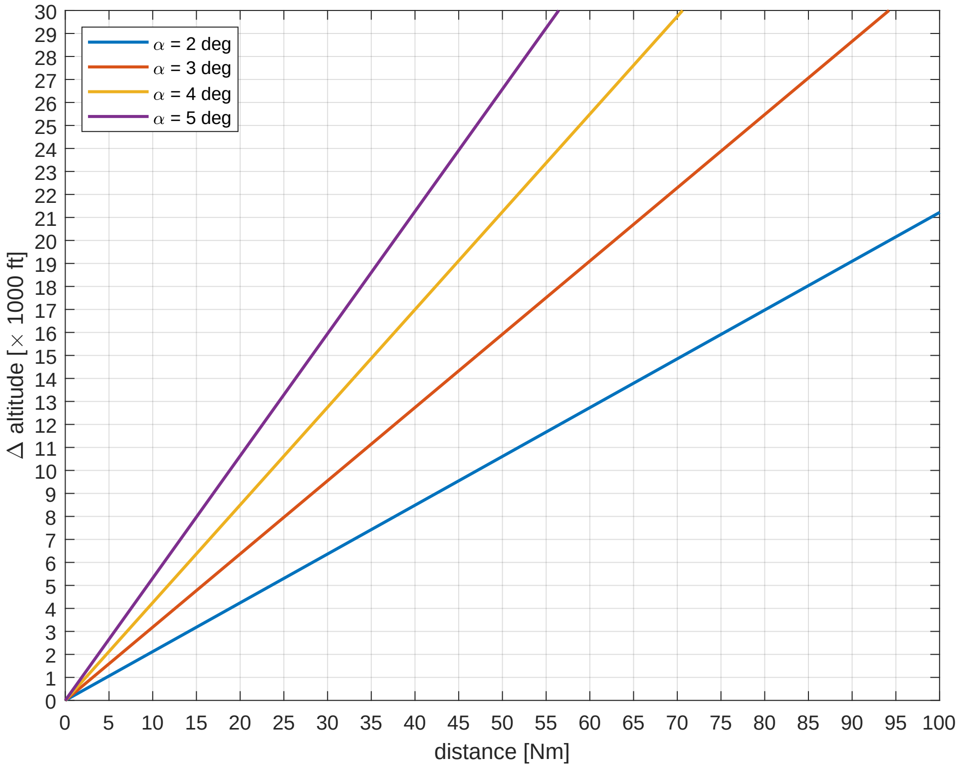

Figure 3: Vertical distance to loose versus lateral distance required for the descent for different glide slope angles.

The glide slope angle which has to be maintained during final approach is indicated on approach charts as illustrated in figure 4. An angle of $3°$ is very common but there are airports like London City Airport with a steeper descent profile.

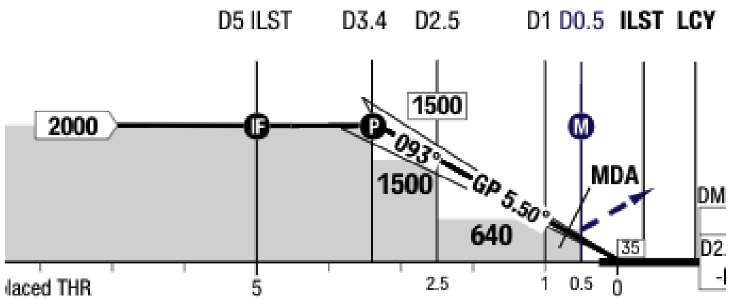

Figure 4: Part of an approach chart indicating the glide path of $5.50°$ during final approach to London City Airport runway 09. (Credit: Lufthansa Systems)

For a $3°$ glide slope, equation (2) leads to a simple rule of thumb:

> The lateral distance (in Nm) required for a given descent on a $3°$ glide slope approximately equals the vertical distance to loose (in ft) divided by 1000 and multiplied by 3.

2.2 Required rate of descent

Beside the lateral distance required for a given descent, a pilot wants to know at what rate the aircraft needs to descend to remain on the desired glide slope. The governing equation is obtained by substituting the distances in equation (1) with their equivalents based on the corresponding speeds. Indeed, if VS is the aircraft's vertical speed and GS its (horizontal) ground speed, then for any time interval Δt:

$$\Delta alt = VS\,\Delta t$$

$$d = GS\,\Delta t$$

Equation (1) then becomes:

$$\tan\,\alpha = \frac{f_2\,VS\,\Delta t}{f_1\,GS\,\Delta t}$$

Factor f_2 is again a unit conversion factor and equals $60\ \mathrm{min/h}$ as vertical speeds are usually expressed as a distance per minute while ground speeds are expressed as a distance per hour.

Rearranging factors yields:

$$\boxed{VS \approx 101.3\,GS\,\tan\,\alpha} \tag{3}$$

The sign of VS is not taken into consideration here. The same formula is applicable for climb and descent profiles, implicitly understanding that the vertical speed on descent is actually negative. For a rigorous mathematical formulation, descent glide slope angles should be given a negative sign.

Equation (3) is plotted in figure 5 with GS as x-axis and VS as y-axis for α ranging from $2°$ to $5°$. By entering the graph via the x-axis with the ground speed and finding the intersection with the line of the desired glide slope angle, the corresponding vertical speed is found on the y-axis.

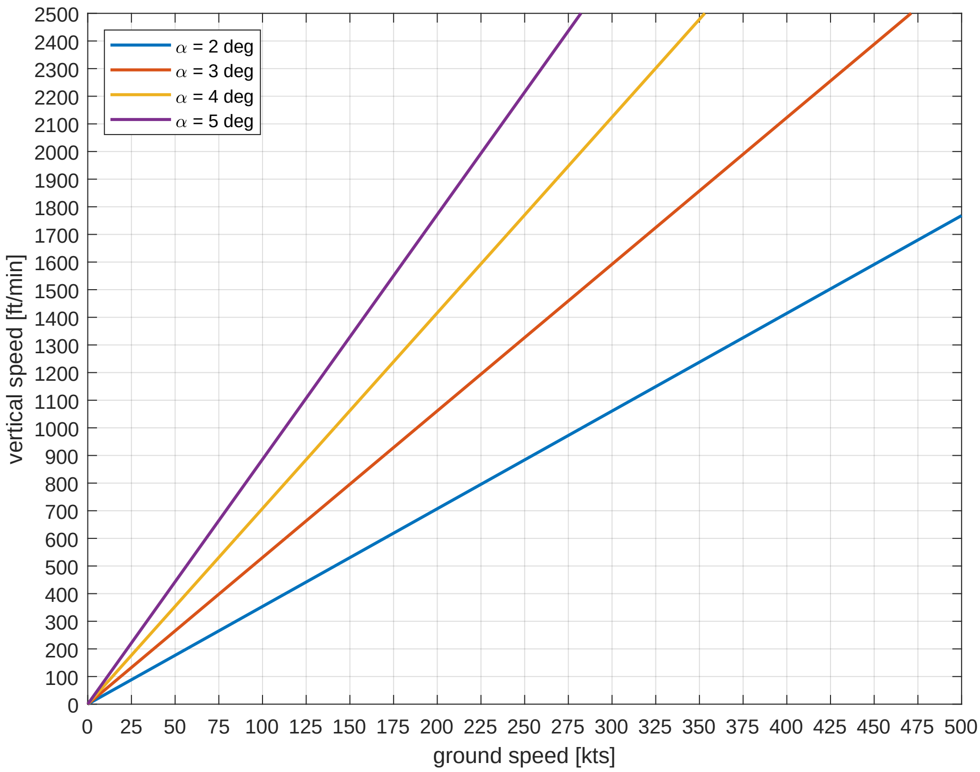

Figure 5: Vertical speed versus ground speed for different glide slope angles.

For a $3°$ glide slope, equation (3) leads to another simple rule of thumb:

> The rate of descent (in $\mathrm{ft/min}$) on a $3°$ glide slope approximately equals the ground speed (in kts) divided by 2 and multiplied by 10.